Paper Reference(s)

## 6664/01

## Edexcel GCE

## Core Mathematics C2

## Gold Level G4

## Time: 1 hour 30 minutes

$\frac{\text { Materials required for examination }}{\text { Mathematical Formulae (Green) }} \quad \frac{\text { Items included with question papers }}{\mathrm{Nil}}$

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

## Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C2), the paper reference (6664), your surname, initials and signature.

## Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 8 questions in this question paper. The total mark for this paper is 75 .

## Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may gain no credit.

Suggested grade boundaries for this paper:

| A* | A | B | C | D | E |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 61 | 53 | 46 | 38 | 31 | 24 |

1. 

$$
\mathrm{f}(x)=2 x^{3}-7 x^{2}-5 x+4
$$

(a) Find the remainder when $\mathrm{f}(x)$ is divided by $(x-1)$.
(b) Use the factor theorem to show that $(x+1)$ is a factor of $\mathrm{f}(x)$.
(c) Factorise $\mathrm{f}(x)$ completely.
2. (a) Find the first 3 terms, in ascending powers of $x$, of the binomial expansion of

$$
(3+b x)^{5}
$$

where $b$ is a non-zero constant. Give each term in its simplest form.

Given that, in this expansion, the coefficient of $x^{2}$ is twice the coefficient of $x$,
(b) find the value of $b$.
3. A company predicts a yearly profit of $£ 120000$ in the year 2013. The company predicts that the yearly profit will rise each year by $5 \%$. The predicted yearly profit forms a geometric sequence with common ratio 1.05 .
(a) Show that the predicted profit in the year 2016 is $£ 138915$.
(b) Find the first year in which the yearly predicted profit exceeds $£ 200000$.
(c) Find the total predicted profit for the years 2013 to 2023 inclusive, giving your answer to the nearest pound.

January 2013
4. (a) Given that

$$
2 \log _{3}(x-5)-\log _{3}(2 x-13)=1
$$

show that $x^{2}-16 x+64=0$.
(b) Hence, or otherwise, solve $2 \log _{3}(x-5)-\log _{3}(2 x-13)=1$.

May 2010
5.


## Figure 1

The triangle $X Y Z$ in Figure 1 has $X Y=6 \mathrm{~cm}, Y Z=9 \mathrm{~cm}, Z X=4 \mathrm{~cm}$ and angle $Z X Y=\alpha$.
The point $W$ lies on the line $X Y$.
The circular arc $Z W$, in Figure 1 is a major arc of the circle with centre $X$ and radius 4 cm .
(a) Show that, to 3 significant figures, $\alpha=2.22$ radians.
(b) Find the area, in $\mathrm{cm}^{2}$, of the major sector $X Z W X$.

The region enclosed by the major arc $Z W$ of the circle and the lines $W Y$ and $Y Z$ is shown shaded in Figure 1.

Calculate
(c) the area of this shaded region,
(d) the perimeter $Z W Y Z$ of this shaded region.
(4)

January 2013
6.


Figure 2
Figure 2 shows a sketch of the circle $C$ with centre $N$ and equation

$$
(x-2)^{2}+(y+1)^{2}=\frac{169}{4} .
$$

(a) Write down the coordinates of $N$.
(b) Find the radius of $C$.

The chord $A B$ of $C$ is parallel to the $x$-axis, lies below the $x$-axis and is of length 12 units as shown in Figure 2.
(c) Find the coordinates of $A$ and the coordinates of $B$.
(d) Show that angle $A N B=134.8^{\circ}$, to the nearest 0.1 of a degree.

The tangents to $C$ at the points $A$ and $B$ meet at the point $P$.
(e) Find the length $A P$, giving your answer to 3 significant figures.
(2)
7. (i) Solve, for $-180^{\circ} \leq x<180^{\circ}$,

$$
\tan \left(x-40^{\circ}\right)=1.5
$$

giving your answers to 1 decimal place.
(ii) (a) Show that the equation

$$
\sin \theta \tan \theta=3 \cos \theta+2
$$

can be written in the form

$$
4 \cos ^{2} \theta+2 \cos \theta-1=0
$$

(b) Hence solve, for $0 \leq \theta<360^{\circ}$,

$$
\sin \theta \tan \theta=3 \cos \theta+2
$$

showing each stage of your working.
8. (a) Sketch, for $0 \leq x \leq 2 \pi$, the graph of $y=\sin \left(x+\frac{\pi}{6}\right)$.
(b) Write down the exact coordinates of the points where the graph meets the coordinate axes.
(c) Solve, for $0 \leq x \leq 2 \pi$, the equation

$$
\sin \left(x+\frac{\pi}{6}\right)=0.65
$$

giving your answers in radians to 2 decimal places.

## END




| Question number | Scheme | Marks |
| :---: | :---: | :---: |
| 4 (a) | $2 \log _{3}(x-5)=\log _{3}(x-5)^{2}$ | B1 |
|  | $\log _{3}(x-5)^{2}-\log _{3}(2 x-13)=\log _{3} \frac{(x-5)^{2}}{2 x-13}$ | M1 |
|  | $\log _{3} 3=1$ seen or used correctly | B1 |
|  | $\log _{3}\left(\frac{P}{Q}\right)=1 \Rightarrow P=3 Q \quad\left\{\frac{(x-5)^{2}}{2 x-13}=3 \quad \Rightarrow \quad(x-5)^{2}=3(2 x-13)\right\}$ | M1 |
|  | $x^{2}-16 x+64=0$ | A1 cso |
|  |  | (5) |
| (b) | $(x-8)(x-8)=0 \quad \Rightarrow \quad x=8 \quad$ Must be seen in part (b). | M1 A1 |
|  | Or: Substitute $x=8$ into original equation and verify. | (2) |
|  | $x=8$ with no working scores both marks. | [7] |

\begin{tabular}{|c|c|c|c|}
\hline Question number \& Scheme \& \& Marks \\
\hline 5 (a) \& \[
\begin{gathered}
9^{2}=4^{2}+6^{2}-2 \times 4 \times 6 \cos \alpha \Rightarrow \cos \alpha=\ldots \\
\cos \alpha=\frac{4^{2}+6^{2}-9^{2}}{2 \times 4 \times 6}\left(=-\frac{29}{48}=-0.604 . .\right) \\
\alpha=2.22 \quad * \\
\text { (NB } \alpha=2.219516005)
\end{gathered}
\] \& \begin{tabular}{l}
Correct use of cosine rule leading to a value for \(\cos \alpha\) \\
Cso (2.22 must be seen here)
\end{tabular} \& M1

A1 <br>

\hline (b) \& \[
$$
\begin{aligned}
& 2 \pi-2.22(=4.06366 \ldots . . .) \\
& \frac{1}{2} \times 4^{2} \times 4.06 " \\
& 32.5
\end{aligned}
$$

\] \& | $2 \pi-2.22$ or awrt 4.06 Correct method for major sector area. |
| :--- |
| Awrt 32.5 | \& | B1 |
| :--- |
| M1 |
| A1 | <br>

\hline \multirow[t]{3}{*}{(c)} \& Area of triangle $=$

\[
\frac{1}{2} \times 4 \times 6 \times \sin 2.22(=9.56)

\] \& Correct expression for the area of triangle $X Y Z$ \& | (3) |
| :--- |
| B1 | <br>

\hline \& \[
$$
\begin{aligned}
& \text { So area required }=" 9.56 "+" 32.5 " \\
& =42.1 \mathrm{~cm}^{2} \text { or } 42.0 \mathrm{~cm}^{2}
\end{aligned}
$$

\] \& | Their triangle $X Y Z+$ (part (b) answer or correct attempt at major sector) |
| :--- |
| Awrt 42.1 or 42.0 (Or just 42) | \& | M1 |
| :--- |
| A1 | <br>


\hline \& Arc length $=4 \times 4.06(=16.24)$ Or $8 \pi-4 \times 2.22$ \& | M1: $4 \times$ their $(2 \pi-2.22)$ |
| :--- |
| Or circumference - minor arc |
| A1: Correct ft expression | \& | (3) |
| :--- |
| M1A1ft | <br>

\hline \multirow[t]{2}{*}{(d)} \& $$
\begin{aligned}
& \text { Perimeter }=Z Y+W Y+\text { Arc Length } \\
& \text { Perimeter }=27.2 \text { or } 27.3
\end{aligned}
$$ \& \[

9+2+Any Arc
\] \& M1 <br>

\hline \& \& \& | (4) |
| :--- |
| [12] | <br>

\hline
\end{tabular}

| Question number | Scheme | Marks |
| :---: | :---: | :---: |
| 6 (a) | $N(2,-1)$ | $\mathrm{B} 1, \mathrm{~B} 1$ <br> (2) |
| (b) | $r=\sqrt{\frac{169}{4}}=\frac{13}{2}=6.5$ | B1 |
|  |  | (1) |
| (c) | Complete Method to find $x$ coordinates, $x_{2}-x_{1}=12$ and $\frac{x_{1}+x_{2}}{2}=2$ | M1 |
|  | then solve to obtain $x_{1}=-4, \quad x_{2}=8$ | A1ft A1ft |
|  | Complete Method to find $y$ coordinates, using equation of circle or Pythagoras i.e. let $d$ be the distance below $N$ of $A$ then $d^{2}=6.5^{2}-6^{2} \Rightarrow d=2.5 \Rightarrow y=. .$ | M1 |
|  | So $y_{2}=y_{1}=-3.5$ | A1 |
|  |  | (5) |
| (d) | Let $\hat{A N} B=2 \theta \Rightarrow \sin \theta=\frac{6}{" 6.5 "} \Rightarrow \theta=(67.38) \ldots$ | M1 |
| (e) | So angle ANB is 134.8 * | A1 |
|  |  | (2) |
|  | $A P$ is perpendicular to $A N$ so using triangle $A N P \tan \theta=$ AP $\qquad$ | M1 |
|  | Therefore $\quad A P=15.6$ | A1cao |
|  |  | (2) |
|  |  | [12] |


| Question number | Scheme | Marks |
| :---: | :---: | :---: |
| 7 (i) | $\begin{aligned} & (\|\alpha\|=56.3099 \ldots) \\ & x=\{\alpha+40=96.309993 \ldots\}=\text { awrt 96.3 } \\ & x-40^{\circ}=-180+" 56.3099 " \ldots \\ & x-40^{\circ}=-\pi+" 0.983 " \ldots \\ & x=\{-180+56.3099 \ldots+40=-83.6901 \ldots\}=\text { awrt } \mathbf{- 8 3 . 7} \end{aligned}$ | B1 <br> M1 <br> A1 <br> (3) |
| (ii)(a) | $\begin{aligned} \sin \theta\left(\frac{\sin \theta}{\cos \theta}\right) & =3 \cos \theta+2 \\ \left(\frac{1-\cos \theta}{\cos \theta}\right) & =3 \cos \theta+2 \\ 1-\cos ^{2} \theta & =3 \cos ^{2} \theta+2 \cos \theta \quad \Rightarrow 0=4 \cos ^{2} \theta+2 \cos \theta-1 * \end{aligned}$ | M1 <br> dM1 <br> A1 cso * <br> (3) |
| (b) | $\cos \theta=\frac{-2 \pm \sqrt{4-4(4)(-1)}}{8}$ <br> or $4\left(\cos \theta \pm \frac{1}{4}\right)^{2} \pm q \pm 1=0, \quad \text { or } \quad\left(2 \cos \theta \pm \frac{1}{2}\right)^{2} \pm q \pm 1=0, q \neq 0 \text { so } \cos \theta=\ldots$ <br> One solution is $72^{\circ}$ or $144^{\circ}$, Two solutions are $72^{\circ}$ and $144^{\circ}$ $\theta=\{72,144,216,288\}$ | M1 <br> A1, A1 <br> M1 A1 (5) <br> [11] |



## Examiner reports

## Question 1

Most candidates attempted this question and many achieved full marks. In part (a), a significant number used long division in order to find the remainder, many successfully but others making sign errors. Those that used the remainder theorem and found $f(1)$ almost always gained full marks.

In part (b), a significant number of candidates gained only one mark as they were able to show that $\mathrm{f}(-1)=0$ successfully but then did not make any comment to the effect that $(x+1)$ was then a factor. Others clearly did not know what was meant by the factor theorem and used long division for which they did not gain any marks.

Part (c) was completed successfully by many candidates. The majority found the quadratic factor by long division rather than inspection of coefficients. Some of those candidates who used a method of long division on occasion arrived at the incorrect quadratic factor because of sign errors. Nearly all candidates who arrived at the correct quadratic factor were then able to factorise it correctly. A number of candidates did not obtain the final mark as they did not write all 3 factors together on one line at the end of their solution.

## Question 2

The most successful strategy seen in part (a) was for candidates to use the formula for $(a+b)^{n}$ to expand $(3+b x)^{5}$ to give $(3)^{5}+{ }^{5} \mathrm{C}_{1}(3)^{4}(b x)+{ }^{5} \mathrm{C}_{2}(3)^{3}(b x)^{2}+\ldots$. A few candidates used Pascal's triangle to correctly derive their binomial coefficients, whilst a few other candidates used $n=3$ in their binomial expansion resulting in incorrect binomial coefficients. A significant number of candidates made a bracketing error to give $270 b x^{2}$ as their term in $x^{2}$. Some candidates did not consider powers of 3 in and wrote $1+5 b x+10 b^{2} x^{2}+\ldots$, whilst a few other candidates did not include any $x$ 's in their binomial expansion.

A minority of candidates wrote $(3+b x)^{5}$ in the form $k\left(1+\frac{b x}{3}\right)^{5}$, and proceeded to apply the $(1+x)^{n}$ form of the binomial expansion. Those candidates who used $k=3^{5}$, usually went on to gain full marks. A significant number of candidates either used $k=1$ or $k=3$ to achieve incorrect answers of either $1+\frac{5 b}{3} x+\frac{10}{9} b^{2} x^{2}+\ldots$ or $3+5 b x+\frac{10}{3} b^{2} x^{2}+\ldots$ respectively and gained only 1 mark for this part.

Part (b) was also fairly well attempted when compared with previous years but there were still a significant number of candidates who did not understand that the coefficient does not include the $x$ or $x^{2}$ part of a term. These candidates were usually unable to form an equation in $b$ alone. A common error was for candidates to form an equation in $b$ by multiplying the coefficient of $x^{2}$ by 2 instead of the coefficient of $x$. A few candidates formed an equation in $b$ using the first and the second terms rather than the second and third terms.

A handful of candidates ignored their binomial expansion and gave the answer 2, using their flawed logic of "twice 2 " being equal to " 2 squared". Other less common errors included either giving an answer of $b=\frac{1}{3}$ following on from $810=270 b$ or $b=1.5$ following on from not multiplying either coefficient by 2 .

## Question 3

Q3(a) was well answered with most candidates gaining the mark for establishing the profit in 2016 correctly. The majority used the $n$th term although some listed the first 4 terms to show the result.

In Q3(b), many candidates adopted a correct approach using logarithms and established a value for their $n$ or $n-1$ but then did not give an answer in the context of the question, i.e. did not use their value of $n$ to establish a calendar year. Those who did go on to find a year were sometimes confused as to which year their value of $n$ implied. A significant number of candidates opted to take a 'trial and improvement' approach by experimenting with different values of $n$. While such methods can gain credit, candidates must be aware that they must show evidence of sufficient work to earn the marks. In this case, examiners would be expecting to see a value of $n$ that gave the year before the profit exceeded $£ 200000$ together with the value of $n$ that gave the year after the profit exceeded $£ 200000$ along with the associated profits. For this kind of approach, if the candidate then went on to identify the correct calendar year, full marks are possible. In this part, some candidates misinterpreted the question as requiring the year when the sum of the profits exceeded $£ 200000$.

In Q3(c), a large number of candidates used the incorrect value of $n$ in the correct sum formula. The use of $n=10$ was the most common incorrect value.

## Question 4

In part (a), while some candidates showed little understanding of the theory of logarithms, others produced excellent solutions. The given answer was probably helpful here, giving confidence in a topic that seems to be demanding at this level. It was important for examiners to see full and correct logarithmic working and incorrect statements such as $\log (x-5)^{2}-\log (2 x-13)=\frac{\log (x-5)^{2}}{\log (2 x-13)}$ were penalised, even when there was apparent 'recovery' (helped by the given answer). The most common reason for failure was the inability to deal with the 1 by using $\log _{3} 3$ or an equivalent approach.

From $\log _{3} \frac{(x-5)^{2}}{(2 x-13)}=1$, it was good to see candidates using the base correctly to obtain $\frac{(x-5)^{2}}{(2 x-13)}=3^{1}$, from which the required equation followed easily.

Even those who were unable to cope with part (a) often managed to understand the link between the parts and solve the quadratic equation correctly in part (b). It was disappointing, however, that some candidates launched into further logarithmic work.

## Question 5

In Q5(a) the majority of candidates could establish the printed angle by using the cosine rule. Some candidates chose to verify that the angle was 2.22 radians by again using the cosine rule to show that $Z Y$ was 9 cm . A small number of candidates worked in degrees and converted to radians at the end.

Q5(b) involved finding the area of the major sector $X Z W X$ but many candidates found the area of the minor sector. As an alternative correct method some candidates found the area of the minor sector and subtracted this from the area of the circle. Some candidates found the area of triangle ZXY and a minority of candidates made some attempt at the area of a segment.

In Q5(c), candidates recognised they needed to find the area of triangle $Z X Y$ and add the area from Q5(b). It was clear here that those with an incorrect Q5(b) did not understand the expression 'major sector' as they were able to score all the marks in Q5(c).

Q5(d) was met with more success although a common error was to add 11 to the minor arc length. Some candidates misinterpreted the perimeter and as a final step, added an attempt at the length $Z W$.

## Question 6

(a) and (b) Most candidates obtained the first three marks for giving the centre and the radius of the circle, but some gave the centre as $(-2,1)$ and a few failed to find the square root of $169 / 4$ and gave 42.25 as the radius.
(c) Diagrams and use of geometry helped some candidates to find the coordinates of $A$ and $B$ quickly and easily. Others used algebraic methods and frequently made sign errors. A common mistake was to put $y=0$ in the equation of the circle. This was not relevant to this question.
(d) Use of the cosine rule on triangle $A N B$ was a neat method to show this result. Others divided triangle $A N B$ into two right angled triangles and obtained an angle from which ANB could be calculated.
(e) This part was frequently omitted and there were some long methods of solution produced by candidates. It was quite common to see candidates obtain equations of lines, coordinates of $P$ and use coordinate geometry to solve this part even though there were only two marks available for this. Simple trigonometry was quicker and less likely to lead to error. $6.5 \times$ tan ANP gave the answer directly.

## Question 7

Although the majority of candidates managed to find the first solution 96.3 in part (i), many struggled with the second solution. Clearly the limits of -180 to +180 were challenging for many candidates, who preferred to give positive answers which were outside the required range. The angle 56.3 was usually found but then it was often subtracted from 180 rather than the other way round. Some candidates, after correctly stating $x-40=56.3$, subtracted 40 to give an answer of 16.3. Just a few thought that $\tan (x-40)$ was equivalent to $\tan x-\tan 40$.

Part (ii)(a) was generally well answered with the correct substitutions made, although there were some instances of incorrect identities such as $\tan \theta=\frac{\cos \theta}{\sin \theta}$ and $\sin \theta=1-\cos \theta$. Mistakes were due more to errors with the basic manipulation of the equation than a lack of knowledge of the identities.

A common mistake came in multiplying the right-hand side by $\cos \theta$ to give $3 \cos ^{2} \theta+2$ instead of $3 \cos ^{2} \theta+2 \cos \theta$.

In part (ii)(b) the quadratic formula was usually quoted and used correctly leading to at least one correct answer $\theta=72$. Those who tried to complete the square often made mistakes, especially in dealing with the coefficient 4 . Candidates who attempted to factorise usually ended up with answers such as 60,90 or 180 and gained no more than one method mark for attempting $360-\theta$. The quadratic formula yielded most success.

Some problems occurred with candidates rounding answers too early and therefore losing accuracy in later steps. Most knew they had to subtract their initial solution from 360 to find other solutions, but some appeared to be randomly adding and subtracting 180, 270 and 360 .

## Question 8

Sketches of the graph of $y=\sin \left(x+\frac{\pi}{6}\right)$ in part (a) were generally disappointing. Although most candidates were awarded a generous method mark for the shape of their graph, many lost the accuracy mark, which required a good sketch for the full domain with features such as turning points, scale and intersections with the axes 'in the right place'. In part (b), the exact coordinates of the points of intersection with the axes were required. Many candidates were clearly uncomfortable working in radians and lost marks through giving their $x$ values in degrees, and those who did use radians sometimes gave rounded decimals instead of exact values. The intersection point $(0,0.5)$ was often omitted.

Part (c) solutions varied considerably in standard from the fully correct to those that began with $\sin \left(x+\frac{\pi}{6}\right)=\sin x+\sin \frac{\pi}{6}=0.65$ The most common mistakes were: failing to include the 'second solution', subtracting from $\pi$ after subtracting $\frac{\pi}{6}$, leaving answers in degrees instead of radians, mixing degrees and radians, and approximating prematurely so that the final answers were insufficiently accurate.

## Statistics for C2 Practice Paper Gold Level G4

Mean score for students achieving grade:

| Qu | Max <br> score | Modal <br> score | Mean <br> $\%$ | ALL | $\mathbf{A}^{*}$ | A | B | C | D | E | $\mathbf{U}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | 8 |  | 80 | 6.40 | 7.77 | 7.56 | 7.26 | 6.88 | 6.33 | 5.52 | 3.34 |
| $\mathbf{2}$ | 6 |  | 74 | 4.46 | 5.82 | 5.70 | 5.22 | 4.66 | 4.04 | 3.34 | 2.00 |
| $\mathbf{3}$ | 9 | 7 | 68 | 6.16 | 8.48 | 7.56 | 6.47 | 5.91 | 5.06 | 4.38 | 2.94 |
| $\mathbf{4}$ | 7 |  | 61 | 4.30 | 6.86 | 6.55 | 5.61 | 4.48 | 3.27 | 2.15 | 0.79 |
| $\mathbf{5}$ | 12 | 12 | 60 | 7.20 | 11.04 | 10.22 | 8.15 | 6.20 | 4.69 | 3.25 | 1.77 |
| $\mathbf{6}$ | 12 |  | 46 | 5.47 |  | 7.95 | 5.20 | 3.91 | 3.19 | 2.52 | 1.41 |
| $\mathbf{7}$ | 11 | 11 | 49 | 5.42 | 10.48 | 9.15 | 7.08 | 5.48 | 4.07 | 2.66 | 0.99 |
| $\mathbf{8}$ | 10 |  | 46 | 4.57 |  | 7.93 | 5.73 | 4.14 | 2.91 | 1.84 | 0.78 |
|  | $\mathbf{7 5}$ |  | $\mathbf{5 8 . 6 4}$ | $\mathbf{4 3 . 9 8}$ | $\mathbf{5 0 . 4 5}$ | $\mathbf{6 2 . 6 2}$ | $\mathbf{5 0 . 7 2}$ | $\mathbf{4 1 . 6 6}$ | $\mathbf{3 3 . 5 6}$ | $\mathbf{2 5 . 6 6}$ | $\mathbf{1 4 . 0 2}$ |

